VIBRATIONS OF POINT SUPPORTED RECTANGULAR THIN PLATES SUBJECTED TO A MOVING FORCE

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1. Introduction

We investigate the dynamic behavior of rectangular thin plate of dimensions $B\ x\ L$ which is simply supported on all edges and has k point supports in its area. Plate is subjected to a force moving with constant speed along one of the edges. Problem is solved by replacing this complex structure with a simply supported plate subjected to a given moving load and k redundant time-varying concentrated forces X_i $(i = 1, \dots, k)$ situated in the positions of intermediate point supports. Vibrations of the plate can be described as:

$$w(x,y,t) = \sum_{i=1}^{k} w^{X_i}(x,y,t) + w^P(x,y,t) \tag{1}$$

where $w^{X_i}(x,y,t)$ are vibrations resulting from the $X_i(t)$ force concentrated in the position (x_i, y_i) of the i intermediate support and $w^P(x,y,t)$ are vibrations of the plate due to a moving force P.

2. Vibrations of the simply supported plate

Equation of motion describing undamped vibration of the simply supported plate have the form:

$$D\left[\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2\frac{\partial^4 w(x,y,t)}{\partial x^2 y^2} + \frac{\partial^4 w(x,y,t)}{\partial x^4}\right] + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t) \tag{2}$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate (E is the Young's modulus and υ is the Poisson's ratio), ρ is the mass density and h is the thickness of the plate. Load function $p(x,y,t)$ has the form $p(x,y,t) = P\delta(x - vt)\delta(y - y_0)$ for the case of force P moving with constant speed v along the axis y_0. For the case of time-varying concentrated force $X_i(t)$ the load function is described as $p(x,y,t) = X_i(t)\delta(x - x_i)\delta(y - y_i)$. Symbol $\delta(.)$ denotes the Dirac delta. Solutions for the analyzed cases have forms:

$$\begin{aligned} w^P(x,y,t) = {} & \frac{4P}{\rho h B L}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{sin\frac{m\pi vt}{L} sin\frac{n\pi y_0}{B} sin\frac{m\pi x}{L} sin\frac{n\pi y}{B}}{\omega_{mn}^2 - \left(\frac{m\pi v}{L}\right)^2} \\ & - \frac{4P}{\rho h B L}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\frac{m\pi v}{L} sin\frac{m\pi vt}{L} sin\frac{n\pi y_0}{B} sin\frac{m\pi x}{L} sin\frac{n\pi y}{B}}{\omega_{mn}\left[\omega_{mn}^2 - \left(\frac{m\pi v}{L}\right)^2\right]} \end{aligned} \tag{3}$$

$$\begin{aligned} & w^{X_i}(x,y,t) = \\ & = \frac{4}{\rho h B L}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} sin\frac{m\pi x_i}{L} sin\frac{n\pi y_i}{B} sin\frac{m\pi x}{L} sin\frac{n\pi y}{B}\int_0^t h_{mn}(t-\tau)X_i(\tau)d\tau \end{aligned} \tag{4}$$

where $\omega_{mn}^2 = \frac{D}{\rho h}\left[\left(\frac{m\pi}{L}\right)^4 + 2\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{B}\right)^2 + \left(\frac{n\pi}{B}\right)^4\right]$ and $h_{mn}(t) = \frac{1}{\omega_{mn}} sin\omega_{mn} t$ is the impulse response function. Since we know that deflections of the point supported plate in the positions (x_i, y_i) are equal zero, forces $X_i(t)$ can be obtained by solving a set of k Volterra Integral Equations:

$$\sum_{i=1}^{k} w^{X_i}(x_i, y_i, t) + w^P(x_i, y_i, t) = 0. \tag{5}$$

3. Numerical example

Presented example is of a rectangular thin plate simply supported on its four edges and point supported at $x_1 = 10\,m;\ y_1 = 10\,m$ and $x_2 = 30\,m;\ y_2 = 10\,m$ (Figure 1). Dimensions of the plate are equal to $B\ x\ L = 20\ x\ 40\ m$ and its thickness $h = 0{,}4\,m$. Plate is subjected to a force of constant magnitude $P = 10\,000\,N$ moving with constant speed $v = 60\,m/s$ along the line $y_0 = 5\,m$. Material properties of the plate are equal to $\rho = 2400\,kg/m^3$, $E = 30\ x\ 10^9\,N/m^2$ and $\upsilon = 0{,}2$. Figure 2 presents vertical vibrations of the point "A" ($x_A = 20\,m;\ y_A = 10\,m$) obtained by using presented method compared with numerical results obtained by using Finite Difference Method. A very good agreement between two methods was observed. Dashed line on the diagram marks the influence line of the static deflection.

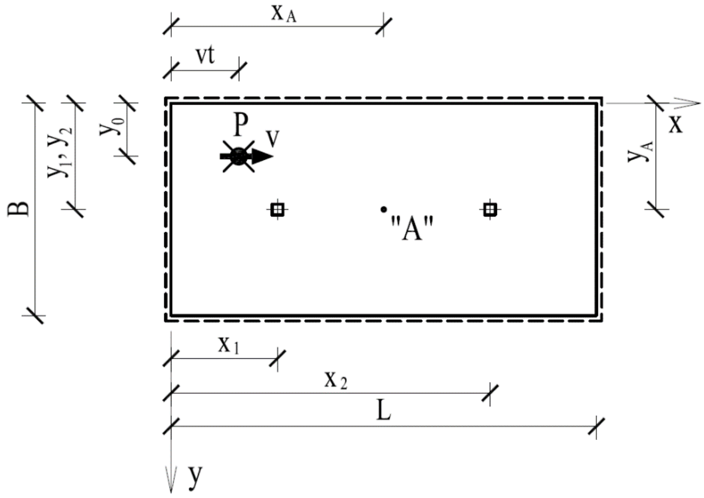

Fig 1. Rectangular thin plate simply supported on all edges with two point supports, subjected to a moving force

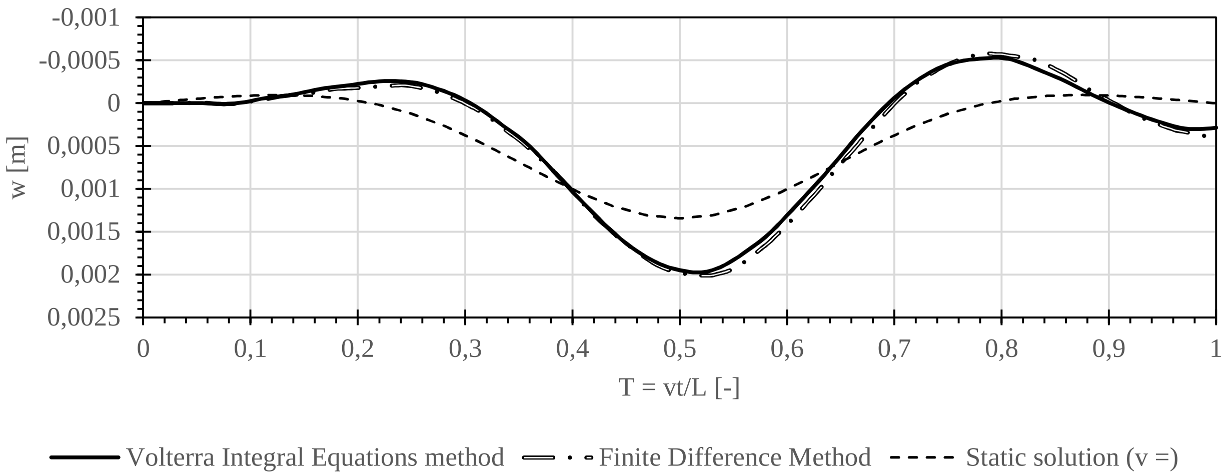

Fig 2. Dynamic deflection of the point "A"

6. References

[1] L. Fryba (1999). *Vibrations of solids and structures under moving load*, Telford, London.

[2] A.W. Leissa (1969). *Vibrations of plates*, US Government Printing Office, Washington DC.

[3] W.Szcześniak (2000). *Wybrane zagadnienia z dynamiki płyt*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa.