EQUILIBRIUM AND STABILITY OF NONLINEARLY ELASTIC CYLINDER FROM BLATZ-KO MATERIAL

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1. General

Stability problem of deformed solid is an important task for both practice and theory. Especially it concerns the structures behavior in form of rods, plates and three-dimensional thin-walled bodies due to the non-linear deformation effects. There are a lot of methods and approaches to have developed in the framework of the three-dimensional theory of elasticity. And one of them is the imposition of a small deformation on the finite one. This approach allows reducing the solution of three-dimensional problem to the homogeneous boundary-value problem, linearized in the small neighborhood of the equilibrium state. In spite of a lot of works devote to description of the thin-walled construction behavior from compressible [1, 5] and incompressible [2] materials the most papers concerns with the simple cases of loading though the buckling effect can also occur under the simultaneous action of several types of loads. On the basic of three-dimensional elasticity theory the stability problem for a hollow finite circular cylinder from Blatz-Ko material subjected to simultaneously several types of loads material is considered here.

To describe the process of stretching and inflation of hollow cylinder with height h, and inner and outer radii r_0 and r_1, correspondingly, the following semi-inverse presentation is used [1, 2]:

$$R = P(r), \Phi = \varphi, Z = \eta z,$$

where R, Φ, Z are the coordinates of material particle after deformation and r, φ, z are the coordinates of this particle in the reference configuration, constant η is a ratio of extension along the direction of the cylinder axis. To describe material properties of compressible nonlinearly elastic media we will use the strain energy function for Blatz-Ko material [3]

$$W = \frac{1}{2}\mu(1-\beta)\left(\frac{I_2}{I_3} + \frac{I_3^{\alpha}-1}{\alpha} - 3\right) + \frac{1}{2}\mu\beta\left(I_1 + \frac{I_3^{-\alpha}-1}{\alpha} - 3\right);$$

where μ is a coefficient and $\alpha = \nu/1-2\nu$, ν is the Poisson's ratio. $I_k = I_k(\mathbf{G}),\ k = 1,2,3$ are the principal invariants of the left Cauchy-Green deformation tensor.

To describe the stress configuration Piola's stress tensor $\mathbf{D}$ is used. The equilibrium equations are given in references configuration by

$$\operatorname{div} \mathbf{D} = 0 \qquad (1)$$

Using the given above explanation the equations (1) reduce to the nonlinear second order differential equation of $P(r)$ function.

If the outer lateral surface is free and the inner lateral surface is loaded then taking (1) into account the boundary conditions take on form

$$\begin{aligned} &e_r \cdot \mathbf{D} = 0, \\ &e_r \cdot \mathbf{D} = -pJ\mathbf{C}^{-1} \cdot e_r, \end{aligned} \qquad (2)$$

p is the coefficient of internal pressure, $\mathbf{C}$ – deformation gradient, J is the determinant of deformation gradient. The cylinder ends conditions are given by the relations

$$D_{zR} = 0, D_{z\Phi} = 0. \quad (3)$$

To study possible bifurcations we consider the small perturbation of prestressed equilibrium state [1, 2]

$$\operatorname{div}\dot{\mathbf{D}} = 0, \dot{\mathbf{D}} = \left[\frac{\partial}{\partial \varepsilon}\mathbf{D}(\mathbf{r} + \varepsilon\mathbf{u})\right]_{\varepsilon=0}, \mathbf{u} = ue_r + ve_\varphi + we_z. \quad (4)$$

$\dot{\mathbf{D}}$ is linearized Piola's stressed tensor, $\mathbf{r}$ is the radius vector in the unloaded state and $\mathbf{u}$ is the perturbation vector.

2. Conclusion

Using the equation (4) the stability problem of hollow elastic finite cylinder was researched within framework of bifurcation approach. By this approach three-dimensional exact equations of Blatz-Ko material for stability analysis have been obtained. To describe the stability area we consider the number of axisymmetric and non-axisymmetric types of deformation. And as a result of research the stability areas of hollow cylinder under tension, compressive and inflation, and also varying the geometric and physical parameters to the above-described material and the forms of cylinder on the edges the stability region have been received.

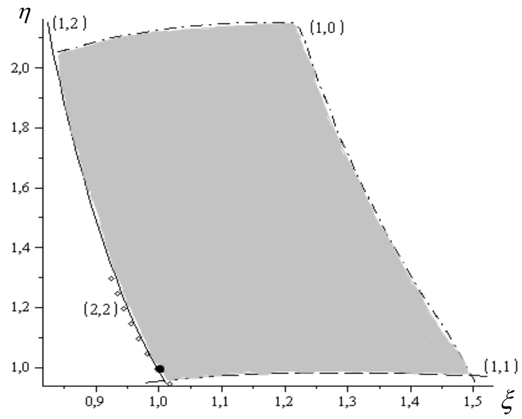

Figure 1. Stability area of the cylinder under tension, compression and inflation with ratios $\alpha = 0.5, \beta = 0, h/r_1 = 10, r_0/r_1 = 0.9$, where $\xi = P(r_1)/r_1$.

3. References

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